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Mioara Mugur-Schächter

Reprinted from FOUNDATIONS OF PHYSICS

Vol. 17, No. 8, August 1987
Printed in Belgium

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Received July 7, 1986

Bell's theorem is believed to establish that the quantum mechanical predictions do not generally admit a causal representation compatible with Einsten's principle of separability, thereby proving incompatibility between quantum mechanics and relativity. This interpretation is contested via two convergent approaches which lead to a sharp distinction between quantum nonseparability and violation of Einstein's theory of relativity.

In a first approach we explicate from the quantum mechanical formalism a concept of "reflected dependence." Founded on this concept, we produce a causal representation of the quantum mechanical probability measure involved in Bell's proof, which is clearly separable in Einstein's sense, i.e., it does not involve supraluminal velocities, and nevertheless is "nonlocal" in Bell's sense. So Bell locality and Einstein separability are distinct *qualifications, and Bell nonlocality (or Bell nonseparability) and Einstein separability are* not *incompatible. It is then proved explicitly that with respect to the mentioned representation Bell's derivation does not hold. So Bell's derivation does not* establish *that* any *Einstein-separable representation is incompatible with quantum mechanics. This first—negative—conclusion is a* syntactic fact.

The characteristics of the representation and of the reasoning involved in the mentioned counterexample to the usual interpretation of Bell's theorem suggest that the representation used—notwithstanding its ability to bring forth the specified syntactic fact—is not factually true. *Factual truth and syntactic properties also have to be radically distinguished in their turn. So, in a second approach, starting from de Broglie's initial relativistic model of a microsystem, a deeper, factually acceptable representation is constructed. The analyses leading to this second representation show that quantum mechanics does indeed involve basically a certain sort of nonseparability, called here de Broglie–Bohr quantum nonseparability. But the de Broglie–Bohr quantum nonseparability is shown to stem directly from the relativistic character of the considerations which led Louis de Broglie to the fundamental relation $p=h/\lambda$,* thereby being essentially consistent with relativity. *As to Einstein separability, it appears to be a still insufficiently specified concept*

[1] Dedicated to Prof. Bernard Grossetête.

[2] Mécanique Quantique et Structures de L'Information, Université de Reims, F51062 Reims Cedex, France.

of which a future, improved specification, will probably be explicitly harmonizable with the de Broglie–Bohr quantum nonseparability.

The ensemble of the conclusions obtained here brings forth a new concept of causality, a concept of folded, zigzag, reflexive causality, *with respect to which the type of causality conceived of up to now appears as a* particular case of outstretched, one-way causality. *The reflexive causality is found compatible with the results of Aspect's experiment, and it suggests new experiments.*

Considered globally, the conclusions obtained in the present work might convert the conceptual situation created by Bell's proof into a process of unification of quantum mechanics and relativity.

1. INTRODUCTION

Bell's proof[1] is founded on a certain mathematical characterization of hypothetical processes of separable individual determination of the observable results produced by a measurement on a microsystem. This mathematical characterization entails an inequality. Certain generalizations[2,3] of this inequality have been studied experimentally.[4–8] The most elaborate[8] among the performed experimental studies is thought by some to eliminate the possibility of a causal theory of the microsystems compatible with Einstein's principle of separability.[9–11] Other authors, on the contrary, hold that the results available so far are not yet conclusive.[12–16] This last opinion, however, is not founded on critiques concerning Bell's theorem itself, but is founded either on rejection[12] of the above-mentioned generalizations[2,3] of Bell's proof or on considerations concerning data rejection[13–16] (the structure of the sample constituted by the registed data is such that, even if the inequality itself is not contested, it is still permitted to conceive that it is only apparently violated by the experimental results)

As to Bell's mathematical representation of Einstein-separable hidden processes of determination of the observed individual results—which is the essence of Bell's proof—it seems to have finally gained unanimous acceptance as the most general one achievable. So it has ceased to be called into discussion anymore.

Furthermore, the assertion that Bell's inequality contradicts indeed Einstein's principle of separability, thereby establishing incompatibility between quantum mechanics and relativity, is not contested.

In this work we produce first a representation different from Bell's one—founded on reflection of (exclusively) the *corpuscular* aspects of a microsystem—which *is* separable in Einstein's sense, i.e., does not involve supraluminal velocities, and nevertheless does *not* fulfill Bell's condition of "locality." So Bell nonlocality does not necessarily entail Einstein nonseparability. Einstein separability and Bell locality are *distinct* qualifications. Furthermore, the mentioned representation cannot be generated from Bell's one by some particularization, while the inverse is possible. So Bell's Einstein-separable representation is *not* the most general one. Now, we prove that with the mentioned Einstein-separable *and* Bell-nonlocal representation, Bell's derivation cannot be performed. So Bell's theorem does not *establish* that *any* Einstein-separable representation is incompatible with the quantum mechanical correlations. This—quite independently of critiques on the generalizations of Bell's theorem or of considerations on data-rejection—is a *syntactic fact* which constitutes a counterexample to the interpretation of Bell's theorem as a proof of incompatibility between quantum mechanics and relativity.

However, this first syntactic result is flawed by a semantic insufficiency. The nature of the representation used and the structure of the argument from the counterexample-proof indicate quite clearly that this representation cannot be factually true, failing to capture the role played in the problem of separability, by the existence also of *wave* aspects of a microsystem. In particular, the representation from the counterexample to the usual interpretation of Bell's theorem is *not connectabe with Aspect's experiments.*

So in the second part of this work we build a deeper representation which encompasses also the wave aspects of the microsystems, and which may come out to be acceptable as being factually true. This improved representation involves a much more complex sort of nonseparability, called here de Broglie–Bohr quantum nonseparability. This sort of nonseparability is connectable with Aspect's results.[8] But it is shown that the de Broglie–Bohr quantum nonseparability cannot be asserted to "violate Einstein's principle of separability." The initial distinction between quantum nonseparability and incompatibility with Einstein's relativity is confirmed. Furthermore, the nature of the distinction is specified in more detail.

The succession of the two mentioned representations constructs progressively a new concept of causality, a concept of *causality folded by reflection, a zigzag, a reflexive causality,* with respect to which the type of causality conceived of up to now appears as *a particular case of outstreched one-way causality.*

Bell's proof is cryptic. It poses directly both the representation used and the quantum mechanical predictional law to which this representation is confronted. Such a procedure offers no references for a critical reexamination. In order to acquire a ground for a critical view, we choose to adopt here an inversed, very progressive procedure: We begin with a detailed study of the content of a quantum mechanical predictional measure. This study reveals the importance of reflections. This leads to the first Einstein-separable and Bell-nonlocal causal representation which offers the formal counterexample to the usual interpretation of Bell's theorem. Unsatisfactory semantical features of this representation send us to an analysis of de Broglie's initial model. This succession almost entails the final representation—nonseparable, causal, and not incompatible with Einstein's relativity—while it also ensures the explicit perception of the various consequences involved by this representation.

2. PROBABILITIES, QUANTUM MECHANICS, OPERATIONS, AND OBSERVATIONS

When physical problems are treated probabilistically, often only the probability measures are defined explicitly and are symbolized. The elementary events and the algebra of events are usually indicated by words only, while quite currently the random phenomenon which produces them remains entirely implicit.

Even in the abstract theory of probabilities—where usually the whole probability space considered is defined explicitly—still, quite currently, there subsists ambiguity concerning the random phenomenon supposed.

The effects of such unachieved specifications can be dominated easily as long as the studied problems do not involve particular conceptual or formal difficulties. But in the case of quantum mechanics such difficulties do amply exist.[17–19] Therefore it becomes imperative to be exhaustively explicit, if it is desired to reach a clear knowledge of the structure of the relationships between the abstract basic probabilistic concepts and the quantum mechanical descriptors, as well as of the physical assertions encoded in this structure. So let us proceed systematically.

In what follows we report results from a preceding work.[20]

2.1. The Abstract Theory of Probabilities

In the abstract theory of probabilities any probability π is defined inside some probability space $[U, \tau, \pi]$, where $U = \{e_i\}$, $i \in I$ (I an index set), is a universe of elementary events e_i, τ is an algebra of events chosen on U, and π is a probability measure posed on τ. Furthermore, the universe U is conceived to be produced by a random phenomenon. But (as we have already remarked) quite currently this supposed random phenomenon is neither defined nor symbolized. Here we shall fill this lacuna.

Let us denote a random phenomenon by (P, U), where P is an "identically" reproducible procedure, each one realization of which brings forth one elementary event $e_i \in U$, in general variable from one realization of P to another one (notwithstanding the supposed identity of the reiterations), whereby U is generated. In order to express that each probability space is tied with some random phenomenon, we shall always consider a *chain* where the defined probability space is preceded by the symbolization of the corresponding random phenomenon:

$$(P, U) \rightarrow\rightarrow\rightarrow [U, \tau, \pi] \tag{1}$$

Without a universe of elementary events, without an algebra of events chosen on this universe, a probability measure simply is not defined, it does not exist. A probability measure alone is not a concept, it is a rag of a concept.

Furthermore, by definition, in the absence of any random phenomenon, a universe of elementary events cannot emerge, hence no space either. So the probability chains (1) are indivisible wholes of the theory of probabilities. One probability chain is the minimal autonomous and closed concept offered by this theory for achieving a probabilistic conceptualization.

But the abstract theory of probabilities does not describe specified phenomena; it only introduces the symbols and defines the calculi with these which characterize any probabilistic conceptualization of phenomena of any nature.

As soon as some specified domain of reality undergoes a probabilistic conceptualization according to the rules of the abstract theory of probabilities, an interpretation of the abstract theory is obtained. Inside this interpretation—unavoidably—some probability chains are conceived of, but where, now, the constituting symbols point, more or less explicitly, toward designata from the described domain of reality. So a particular semantics comes in.

Quantum mechanics is a probabilistic description of microsystems. So what are the probability spaces specific of quantum mechanics, carrying the corresponding semantics? And what is the code which connects the fundamental abstract probabilistic symbols P, U, τ, π, with the fundamental descriptors of quantum mechanics?

2.2. One-System Quantum Mechanics

Algorismic Chains. Consider a pair $(|\psi_1\rangle, A)$ where $|\psi_1\rangle = |\psi(t_1)\rangle$ is the state vector assigned at the time t_1 to the considered microsystem S, and A is a Hermitian operator representing a dynamical observable—in the mathematical sense—defined for S. For each such pair an algorismic chain can be constructed inside quantum mechanics, namely

$$(|\psi_1\rangle, A) \to\to\to [a_J, \tau_A, \pi(\psi_1, A)] \qquad (1')$$

where the universe of elementary events $a_J = \{a_j, j \in J\}$ (J an index set) is the spectrum of A determined by the equation $A|u_j\rangle = a_j|u_j\rangle$ for eigenvectors and eigenvalues of A; τ_A is the total algebra on a_J; finally, $\pi(\psi_1, A)$ is the probability density posed on τ_A, calculated—via the law of total probabilities—from the well-known quantum mechanical elementary probability density corresponding to $|\psi_1\rangle$ and A, namely, for any $j \in J$,

$$\pi(\psi_1\rangle, a_J) = |\langle u_j | \psi_1\rangle|^2 \qquad (2)$$

(for simplicity we suppose a nondegenerate situation).

Observational Probability Spaces. The algorismic space from (1′) is only a coded representation of another, observable space,

$$[V_A(t_2, d_A), \tau_A, \pi(P, t_0, t_1, E, A)] \qquad (3)$$

where A designates an observable and numerically valued aspect *of a macroscopic device* D_A, "the needle position of D_A"; $V_A(t_2, d_A) = \{V_j, j \in J\}$ is the universe of the possible values of A, brought forth by reiterations of a measurement process $M_A(t_1, t_2, d_A)$, a process which is accomplished in a spatial domain d_A and which begins at a time t_1 when the state vector of S is $|\psi_1\rangle$ and then lasts for some time interval $(t_2 - t_1) > 0$; τ_A is the total algebra on $V_A(t_2, d_A)$; $\pi(P, t_0, t_1, E, A)$ is the probability measure put on τ_A, depending on t_1, A, as well as on the initial time t_0, on the preparation procedure P used for creating the state $|\psi_0\rangle$ on the transform of which at t_1 the measurement M_A begins, and—finally—on the evolution law E of S during $t_1 - t_0$.

Like any probability measure, $\pi(P, t_0, t_1, E, A)$ is determined via the law of total probabilities by the density $\pi(P, t_0, t_1, E, V_j)$ for the elementary events $V_j \in V_A(t_2, d_A)$.

In the writing of the observable space (3) all the space-time dependences have been explicitly stated and symbolized.

Now, each observable value is posed to be numerically equal to a corresponding eigenvalue $a_j \in a_J$ labelled by the same index $j \in J$:

$$(V_j \in V(t_2, d_A)) = (a_j \in a_J) \qquad (4)$$

Furthermore, each observable probability density $\pi(P, t_0, t_1, E, V_j)$ is posed to be numerically equal to the corresponding algorismic probability density, i.e., for any $|\psi\rangle$ and any $j \in J$

$$\pi(P, t_0, t_1, E, V_j\rangle = |\langle u_j | \psi_1 \rangle|^2 \tag{5}$$

In *this* sense the calculated probability density (2) is a "predictional law," verifiable by the help of the relative frequencies of the observed values V_j, at the limit of large numbers.

So (4) and (5) form the key of the code which translates the algorismic space from (1) into the observable space (3).

The Random Phenomena. The dependence on the operation P, the moments t_0, t_1, and the evolution E, of the observable probability measure from (3)—a space founded on a universe which emerges at a time t_2 *ulterior* to these moments, this operation, and this evolution—expresses clearly the fact that the space (3) is not a self-sufficient concept. This concept calls for a closure where all these specifications to which it refers be contained: it calls for the definition of the corresponding random phenomenon.

As soon as it is explicitly researched, the definition of this random phenomenon can be easily constructed, but it is particularly complex. Indeed this definition brings in a sequence of three partial procedures which cover three distinct space-time domains.

The first partial procedure is a preparation operation $P(\Delta t_0, d_P)$ covering some nonnull space-time domain $d_P \times \Delta t_0$ and which at its final moment t_0 introduces a state of S representable by an initial state vector $|\psi(t_0)\rangle = |\psi_0\rangle$.

The second partial procedure—which does not necessarily exist—is an evolution $E(t_0, t_1, d_C)$ of the initial state of S, preceding the measurement performed on S and covering some space-time interval $d_C \times (t_1 - t_2)$, where the index C stays for the external conditions, fields and obstacles, which determine the considered evolution. If H denotes the Hamiltonian corresponding to the conditions C and $T(t_0, t_1, H)$ is the respective propagator, then $|\psi_0\rangle$ is such that $|\psi_1\rangle = T(t_0, t_1, H)|\psi_0\rangle$ is precisely the state for which is defined the observable probability space (3).

Finally, the third partial procedure is the measurement operation $M_A(t_1, t_2, d_A)$ from the definition of (3).

As soon as the time $t_1 \geqslant t_0$ is fixed, the succession

$$[P(\Delta t_0, d_P), E(t_0, t_1, d_C), M(t_1, t_2, d_A)] = P(P, t_0, t_1, E, A) \tag{6}$$

becomes "one identically reproducible procedure P," each reiteration of P reestablishing the origin of time t_0.

Each realization of this stable procedure brings forth *one*, V_j, among all the various elementary events from the universe of elementary events $U = V_A$. Thus we are in the presence of a random phenomenon (P, U) in the standard sense of the term, namely

$$(P, U) = ([P(\Delta t_0, d_P), E(t_0, t_1, d_C), M_A(t_1, t_2, d_A)], V_A(t_2, d_A)) \quad (7)$$

This is the researched definition, labelled by all the relevant space-time specifications.

The "one" random phenomenon (7) brings in three different space-time domains, namely $d_P \times \Delta t_0$ where the preparation process occurs, $d_C \times (t_1 - t_0)$ where the evolution E occurs, and $d_A \times (t_2 - t_1)$ where the measurement process M_A is performed.

The Chains. We are now finally able to write down the chains (1) which, by their semantic content, characterize the particular probabilistic description named quantum mechanics:

To each algorismic chain

$$(|\psi_1\rangle, A) \to\to\to (a_J, \tau_A, \pi(\psi_1, A)) \quad (1')$$

there corresponds an operational-observational probability chain

$$\begin{aligned}&([P(\Delta t_0, d_P), E(t_0, t_1, d_C), M_A(t_1, t_2, d_A)], V_A(t_2, d_A))\\ &\quad \to\to\to (V_A(t_2, d_A), \tau_A, \pi(P, t_0, t_1, E, A)) \quad (8)\end{aligned}$$

The time t_1 in (8) is fixed but arbitrary, so the Schrödinger evolution of the probability law π is incorporated in (8).

The operational-observational probability chains (8) are the basic conceptual wholes of quantum mechanics. Any quantum mechanical prediction belongs to some algorismic chain (1′), which translates into a corresponding chain (8).

The ensemble of the expressions (1′) → (8), imported here from a preceding work,[20] yields the researched explication of the structure of the relations between the basic probabilistic concepts P, U, τ, π, and the quantum mechanical formal descriptors. It also elucidates the semantical, the physical content, assigned by the quantum mechanical description to the basic probabilistic concepts. This enables us now to make a crucial remark concerning, in particular, the quantum mechanical predictional laws.

Physical Content of a Quantum Mechanical Measure. The quantum mechanical probability measures $\pi(P, t_0, t_1, E, A)$ are *defined* as the *global* result of the *composed* effects produced, on the one hand, by the preparation P and the evolution E, i.e., by the object state, *and* on the other hand by the processes and devices involved in the measurement process M_A. The contributions from these two different sources are *not* mutually individualized inside a given prediction π, as happens in the classical probabilistic theories. Bohr and Heisenberg have strongly stressed this fundamental aspect of the quantum mechanical description, but in an unformalized way, and without distinguishing explicitly and radically between preparation and measurement. Here both this global aspect and all the distinctions involved came into evidence as a result of a formalized analysis.

The significance of Bell's inequality will appear to be intimately connected with this unanalyzed character of the quantum mechanical predictional measures.

2.3. Composed-System Quantum Mechanics

The preceding approach admits a straightforward generalization to the case of a system consisting of two or more components. However, in this case the well-known tensor products formalism is used. This formalism associates a state vector only to the global system, while the component systems are devoid of an individualized descriptor. In consequence of this, the analogs for a composed system, of the expressions $(1') \rightarrow (8)$, yield still *less* individualized information concerning the contributions to the observable results of measurements, of the various physical designata involved, microobjects, operations, processes.

The problem studied in Bell's proof is placed precisely inside this zone of increased shadow.

2.4. Quantum Mechanical Measurement Theory

In the object quantum mechanics the measurement processes are only posed to exist but they are not described. So it might be hoped that in the quantum mechanical theory of measurements, where the measurements themselves are studied, the absence of mutual distinction between the effects on the observed distributions, of the preparation, the eventual evolution which precedes the measurement, and the measurement, is somehow suppressed. But in fact this does not happen. The quantum mechanical description of a measurement process is a metadescription where the measuring device involved D_A is treated as a component of a new composed object system $S + D_A$. This is described by the help of the tensor products formalism, which, for any object-system, not composed or composed, leads always again to *less* informative forms $(1') \rightarrow (8)$ than does the corresponding object-description.

2.5. Partial Conclusion

The quantum mechanical formalism yields no explicit information on the *isolated* contibutions to the distributions of observed results, of the object-system state, and of the devices and processes involved in the measurement procedure. If such information is implicitly stored in the formalism, only some adequate indirect analyses could reveal it.

3. REFLECTED DEPENDENCE

So let us attempt an indirect analysis in order to obtain information concerning the particular structure of the dependences on the various factors involved in a spin measurement on pairs.

3.1. A One-System Investigation on Spin-Measurements on Pairs

Consider the experimental arrangement indicated in Fig. 1, similar to that in Bell's proof. Here σ indicates a source of pairs $(S_1 + S_2)$, each pair constituting a composed system; D_{1A} and D_{2A} indicate, respectively, the ensemble of the measuring devices wich interact with the subsystem 1 and the ensemble of the measuring devices which interact with the subsystem 2; F_1 and C_1 are, respectively, the Stern–Gerlach field and the counter from D_{1A}, while F_2 and C_2 are respectively the Stern–Gerlach field and the counter from D_{2A}; θ_1 and θ_2 are the angles which characterize the directions of the inhomogeneous Stern–Gerlach fields F_1 and F_2, respectively; l_1, l_2 are the breadths, respectively, of the spatial domains where F_1 and F_2 are realized and C symbolizes a coincidence-register. The preparation

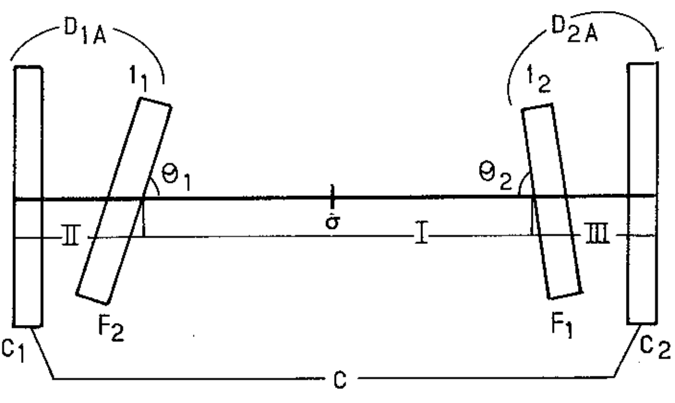

Fig. 1. Experimental arrangement similar to that in Bell's proof.

process $P(\Delta t_0, d_P)$ and the evolution $E(t_0, t_1, d_C)$ take place in the spatial domain I, while the spatial domains II and III, respectively, are occupied by the devices D_{1A} and D_{2A}, and there the measurement processes take place during $(t_2 - t_1)$ (with respect to the reiterated origin of times t_0) (Fig. 2).

Throughout what follows we place ourselves inside the standard Newtonian quantum mechanics concerning massive microsystems, and we consider exclusively a stationary situation. *Photons and flipping devices are for the moment excluded.* So the reader must for the moment restrain severely from any mental confrontation with Aspect's experiments.

We choose now to ask the question: What happens with the designatum towards which point the words "the components S_1 and S_2 of the created pair $(S_1 + S_2)$," inside the space-time domain $d \times (t_2 - t_0)$ of which the spatial extension covers the whole region $d = \text{I} + \text{II} + \text{III}$, while the time extension $(t_2 - t_0)$ is that which follows the preparation process of the pair and includes the durations of both the evolution E and the measurement M_A?

An answer to this question cannot be obtained by the help of the quantum theory of measurements, nor by the use of the quantum theory of composed systems, since the tensor products formalism de-individualizes the description of the component systems. So, as announced, we shall research indirect specifications by the use of the one-system quantum theory. Namely, we imagine that the source σ prepares only *one* microsystem S instead of a pair, and we transpose to this microsystem the question formulated above.

Consider then first the case $\theta_1 = \theta_2 = \pi/2$ and suppose that the

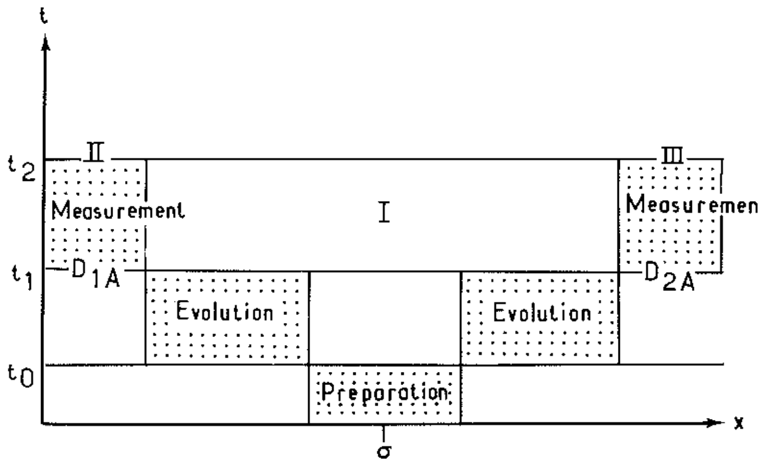

Fig. 2. Space-time specifications for the experiment represented in Fig. 1.

prepared system S is first emitted toward F_1. What happens then with S in the presence of the devices from Fig. 1? According to the one-system Newtonian quantum mechanics, the fields F_1 and F_2 are to be assimilated to two potential barriers. Indeed each field introduces a non-null index of refraction which is equivalent to a repulsive potential acting inside the finite spatial domain occupied by that field. (Though orthogonally to ox the acting Stern–Gerlach fields are inhomogeneous, these fields can be considered to be quasi-constant along the directions of propagation: Since we study only hints concerning the dependences on the various factors involved, gross considerations are sufficient.) Furthermore, in general a fraction of the systems S which are incident on a Stern–Gerlach field will propagate beyond field, i.e., in general the counters will be activated, exactly as in a spin measurement process. So—in essence—we are in the presence of an elementary square potentials problem, but which leads to registrations of spin-values. With θ_1 and θ_2 parallel and perpendicular on the direction of propagation, this problem can be treated in only one dimension.

With these assumptions the general solution of the Schrödinger equation of S in I, II, and III, and during $(t_2 - t_0)$, is accessible via the well-known standard methods:

In general a potential barrier introduces a non-null reflection coefficient $R \neq 0$, corresponding to an index of transmission $T = 1 - R$. A barrier can reach the limiting character $R = 0$, $T = 1$, corresponding to a perfect transmission (diffusion resonance) only in very particular conditions. Let us suppose that the field F_1 introduces a reflection coefficient $R_1 \neq 0$. Then in I the linearity of the formalism and the fact that the Hamiltonian is degenerate with respect to the eigenvalues of the momentum operator entail that the general form of the state vector of S_1 is a superposition

$$|\Psi(\mathbf{r}, t)\rangle = a\,|\Psi_i(\mathbf{r}_i, t) + b\,|\Psi_\rho(\mathbf{r}_\rho, t) \tag{9}$$

$$\mathbf{r}_i \in I - d_\rho, \qquad \mathbf{r}_\rho \in I, \qquad t \in (t_2 - t_0)$$

of a packet $a|\Psi_i\rangle$ incident on the field F_1, defined only in the part of I where the preparation process does not penetrate, *and* a reflected packet $b|\Psi_\rho\rangle$ defined throughout I, i.e., which is incident on the field F_2, the coefficients a and b being determined by R_1, T_1 (this reflected term is null only in the case $T_1 = 1$, of resonance diffusion).

Now, F_2 also introduces in general a nonnull index of reflection $R_2 \neq 0$, which in particular can reach the limit $R_2 = 1$. But $R_2 \neq 0$ entails $T_2 \neq 0$. So when the reflected term from (9) does exist, at least a part of it can be transmitted in III and can reach the counter C_2. This transmitted part of the reflected term from (9) carries to C_2 *information* on the field F_1, on its orientation, its form and dimensions, and its index of reflection, *before* the time t_2 when an observed value $V_j \in V_A$ emerges.

Mutatis mutandis, the same conclusion holds for a system S which propagates towards F_2. The state of S is found to penetrate in general inside the domain II and to carry to the counter C_1 information concerning the field F_2, before the time t_2 when a registration by C_1 can arise.

Consider now the general case of an arbitrary pair of values θ_1 and θ_2. The treatment ceases to be one-dimensional. The existence in II at the counter C_1 of a contribution from a state of S which was emitted toward F_2, and vice versa for III at C_2 and a state of S emitted toward F_1, depends now on the particular pair of values chosen for the angles θ_1 and θ_2 (and also—as before—on the dimensions and the forms of the spatial domains F_1 and F_2, the distance between them, their indexes of reflection). Furthermore, in the case $\theta_1 = \theta_2 = \pi/2$ the registrations obtained via C_1 are devoid of information concerning F_2 only if $R_2 = 0$, and the registrations via C_2 are devoid of information concerning F_1 only if $R_1 = 0$. While with $\theta_1 \neq \theta_2$ it can happen that the registrations via C_1 be devoid of information concerning F_2 even if $R_2 \neq 0$: It suffices that the intersection between the spatial domain covered by F_1, and the geometric shadow of F_2, be zero, and vice versa for C_2 and F_1. But, in *general*, one or the other of these crossed, reflected informations, or both, do exist.

The above examination of a one-system evolution in the conditions characteristic of a spin measurement on pairs discloses an interesting indication:

A microsystem which evolves under the external conditions corresponding to a spin measurement on pairs produces observables results involving the existence—in general—of a crossed dependence, on the Stern–Gerlach field F_2, of the results obtained via the partial device D_{1A}, and on the field F_1, of the results obtained via the partial device D_{2A}. These crossed dependences stem from the fact that the fields and the material devices act also as obstacles.

The linearity of the quantum mechanical formalism and the quantum mechanical coefficients of reflection and transmission offer a natural way for expressing this fact mathematically.

In the light of the indication obtained above it seems clear that the quantum mechanical predictional measure concerning spin measurements on pairs equally involves the same type of crossed dependences. On the basis of the preceding formal definitions—that the mean value of the products of simultaneous outcomes on the two devices involved in the experiment from Bell's proof is $\langle \boldsymbol{\sigma}_1 \cdot \mathbf{u}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{u}_2 \rangle = -\mathbf{u}_1 \cdot \mathbf{u}_2$, where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the spin operators defined respectively for S_1 and S_2 and $\mathbf{u}_1$ and $\mathbf{u}_2$ are the unit vectors of the directions of F_1 and F_2 respectively. But this is only a *global* assertion where all the more detailed specifications of the type of those brought forth by the preceding analyses concerning only one system S are washed out by the normalization of the reflecting surfaces of F_1 and F_2 and by an integration over the physical-space variables, operated on the well-known complete spinor representation.[21] Therefore the crossed dependences due to reflections on F_1 and F_2 are implicit. The analogous assertion holds concerning the temporal structure of the developments by which the reflected dependences are produced. The quantum mechanical prediction yields no description of it. When a stationary situation is considered, the *final* result is asserted directly.

But beyond this particular conclusion concerning spin measurements on pairs, the preceding considerations disclose a much more general fact. This fact will now be expressed in the form of a principle which completes Heisenberg's principle of uncertainty.

3.2. The Principle of Reflected Dependence

The preparation and the measurement devices produce both diffractions and reflections of the object-system state.

When two successive measurements of two noncommuting observables, performed on one same object-system state, are considered, the diffractions entail a certain limitation concerning the dispersions of the two observed distributions. This limitation has been explicitly expressed, first outside the formalism of quantum mechanics, by the uncertainty principle, and then also by the derivation of uncertainty relations inside the quantum mechanical formalism.

On the contrary, the observable effects produced by reflections of the object-system state have not been explicitly examined so far, even though the formalism does assert them implicitly. But the preceding analyses bring now into evidence a quite general explicit assertion:

As soon as a measurement of an observable A involves several sets of measuring devices D_{iA}, $i=2, 3,\ldots$, if one of these sets, D_{kA}, produces by reflection of the object-system state a geometric shadow which intersects the spatial domain occupied by another set D_{qA}, the probability for registering via D_{qA} a value V_j of A depends in general on characters of D_{kA}.

The above statement can be considered to pose a *principle of reflected dependence*. Indeed the statement holds quite generally, for measurements on a composed system with two, three, or more components, as well as for measurements on a *one*-system state but which consists of a superposition of two or several quasi-disjoint packets (like those created in interferometry by the use of an amplitude divisor), therefore requiring more then one set of devices for performing a measurement.[22] Of course, when the duration assigned to the measurement interaction is practically zero (as in the case of a position measurement), the reflected dependence might not arise, even if the formulated geometric condition is fulfilled.

While the uncertainty principle admitted a natural, relatively simple quantum mechanical expression, the principle of reflected dependence opposes resistance to a quantum mechanical expression. This, no doubt, is the reason why it remained implicit. Indeed, quantum mechanics contains no representation of the "characters of D_{qA}." Therefore one is led to study a detour. For instance, the simultaneous registrations of values V_{ij} via two or more sets of devices D_{iA}—"coincidences"—might seem at first sight to be a convenient concept for achieving a formal quantum mechanical expression of the principle of reflected dependence: The assertion of the existence of reflected dependence entails that the probability of a coincidence of two or several registrations of values V_{ij} is different from the product of the probabilities of these registrations considered separately. This is already a mathematical formulation, namely a probabilistic one. However, it is not yet a quantum mechanical formulation. The quantum mechanical formalism does not permit a direct translation of the above probabilistic formulation. This is so because the formalism does not define individualized probabilities for the registrations via *partial* sets of devices, so it permits one to treat the coincidences only *globally*, i.e., with respect to the integral probability measure, corresponding to the integral state vector and to the *ensemble* of all the apparatuses involved in the considered measurement, undistinguished from one another. So, in fact, a quantum mechanical formulation cannot be realized. The one-system theory, as it appeared above, permits one only to become aware of the physical as well as of the formal (implicit) presence of reflected dependences, but it cannot yield an explicit formal representation of these.

The uncertaity principle and the principle of reflected dependence are of the same essence. They both concern unavoidable effects on the observable distributions, produced by the interaction between macroscopic devices and a microscopic object-system—effects where the contributions of the object-system are not distinguished—inside the quantum mechanical predictions—from those of the macroscopic devices. Both these assertions are profoundly tied to Bohr's views on the relations between microobjects, macroobjects, and knowledge. The principle of reflected dependence *completes* the uncertainty principle for the characterization of these relation.

4. BELL'S THEOREM

According to Einstein's principle of separability "the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former."[23]

We shall now build an obviously Einstein-separable causal representation of hypothetical hidden processes of determination of the individual observable results produced by a spin measurement on pairs. This representation is founded on the concept of reflected dependence. Though the representation *is* Einstein-separable, it nevertheless modelizes the *existence* of the crossed dependences qualified as "nonlocal" by Bell, and which therefore were banished form his representation. So the representation built here is both Einstein-separable and Bell-nonlocal. Its structure is such that it does no permit Bell's derivation. Thereby it shows—by a counterexample to the usual interpretation—that

Bell's proof does not establish that any Einstein-separable representation is incompatible with quantum mechanics.

Obviously, the syntactic producibility of this negative proposition, by itself, entails strictly nothing concerning the factual existence or inexistence of supraluminal influences, nor concerning the factual truth of the separable representation used here. But *other* reasons *eliminate* the possibility of this truth. The outrageously classical character of the representation, joined to certain features of the counterexample-proof, indicates quite clearly that *this representation cannot be factually true.* So the qualification of the factual situation will have to be obtained by some subsequent examinations.

Qualitative Approach. We consider again specifically the measurement involved in Bell's proof. We still remain inside Newtonian quantum mechanics, as Bell's proof—*stricto sensu*—requires.

Consider again the analysis from Sec. 3.1. The quantum mechanical reflection and transmission coefficients concern the capacity of producing observable *localized*, "corpuscular"-like effects (impacts on sensitive screens, etc.). Suppose first the general case where the reflection coefficients R_1 and R_2 introduced by the fields F_1 and F_2, respectively, are both nonnull. This nonnullity is a statistical assertion. In individual terms it means that for one subsystem S_1 from one pair $S = (S_1 + S_2)$, which begins by being incident on the field F_1, the capacity of producing observable localized effects is either transmitted, or reflected. If it is reflected, and if the geometric shadow of the spatial support of F_1 does intersect the domain III occupied by the devices D_{2A}, then this subsystem S_1 can act observably on the counter C_2. *Mutatis mutandis*, the same assertion holds for the subsystem S_2 from the considered pair.

This has consequences concerning the possible hidden compositions of a coincidence. Namely: a coincidence can be produced, either via the *direct* transmissions of the capacities of corpuscular effects of a subsystem S_1 incident on F_1 and of a subsystem S_2 incident on F_2, in which case S_1 and S_2 must belong to the *same* generation; or it can be due to one direct transmission, and one reflection produced for a subsystem from a *preceding* generation; or it can even be due to two reflections (for subsystems *belonging* or *not* to the same generation).

Now, the last two sorts of compositions suggest the hope that it might be possible to build—founded on the explicit consideration of reflections—a trivially separable hidden representation of the quantum mechanical measure. Indeed, as soon as the notion of coincidences tied either to two *different* generations of pairs or to *two-reflections* in one generation, has been conceived, it permits one to understand the vehiculation of the cross-information between D_{1A} and D_{2A} stored in the correlations asserted by quantum mechanics, *without any constraints on velocities.*

Definitions, Notations, Dependences. We label by $n = 0, 1, 2..., \infty$ the successive emissions of a pair, $S_n = (S_{1n} + S_{2n})$, with 0 marking an arbitrary initial time.

Let ${}^i\lambda_{1n}$ and ${}^i\lambda_{2n}$ symbolize any structure of qualifications added to the quantum mechanical ones and characterizing individually the emitted states of S_{1n} and S_{2n} respectively, as well as their evolutions before eventually a reflection takes place, i.e., their states wich are directly incident on F_1 and F_2 respectively.

Let ${}^\rho\lambda_{1n}$ and ${}^\rho\lambda_{2n}$ symbolize any structure of qualifications characterizing individually a state of S_{1n} produced by reflection on F_1, and a state of S_{2n} produced by reflection on F_2, respectively, as well as the evolutions of these states until they eventually arrive now incident on—inversely—F_2 or F_1, respectively.

We now admit by a first hypothesis H_1 that

H_1. *No supraluminal velocities do occur.*

We also introduce—tentatively but explicitly—a second hypothesis H_2 which specifies the connection assumed here with the quantum mechanical formalism. Namely:

H_2. $R_1 = R_2 = 0$ *entails that no reflected individual states* ${}^\rho\lambda$ *do exist.*

The hypothesis H_2 restricts the role of the designata of the symbols ${}^\rho\lambda$. It amounts to the supposition that these designata act *exclusively* as the individual elements constituting the substratum of the quantum mechanical reflection coefficients R_1 and R_2. These—by definition—concern *only* the reflection of the capacity of the described microsystem to produce localized, *corpuscular*-like observable effects. So the hypothesis H_2 excludes from consideration any effects of eventual reflections of pure wave aspects of the microsystems. Obviously, this is a strongly simplifying restriction. But this simplification will prove useful for our present purpose, namely, for distinguishing radically between Einstein-separability and inexistence of crossed dependences.

Let us now classify the possible coincidences.

Let $C(i, i, n, n)$ denote a coincidence produced by transmission inside the fields F_1 and F_2 of a pair $({}^i\lambda_{1n}, {}^i\lambda_{2n})$ of two directly incident subsystem-states, necessarily stemming from the same generation S_n. This coincidence is numerically representable by the product of the corresponding simultaneously observed values. The first value from this coincidence, V_{1j}, is registered via interactions between the object-state ${}^i\lambda_{1n}$ which triggers it and the set of devices D_{1A}, so it depends on characteristics of the field F_1 and on ${}^i\lambda_{1n}$. Similarly, the value V_{2j}, depends on the field F_2 and on the state ${}^i\lambda_{2n}$.

Now, in order to ensure an uncontestable consistency with H_1, we admit by a third hypothesis H_3 that

H_3. The values V_{1j} and V_{2j}, which are triggered by directly incident states ${}^i\lambda_{1n}$ and ${}^i\lambda_{2n}$, respectively, do not involve crossed dependences. So we write

$$[V_{1j}(F_1, {}^i\lambda_{1n})\text{: ind. } F_2;\ \ V_{2j}(F_2, {}^i\lambda_{2n})\text{: ind. } F_1)] \tag{10}$$

(ind.: independent)

This entails that in a coincidence $C(i, i, n, n)$ the value V_{1j} is "uninformed" of F_2 and, vice versa, the value V_{2j}, is uninformed of F_1. Consequently the dependences are $C(i, i, n, n) = [V_{1j}(F_1, {}^i\lambda_{1n}) \cdot V_{2j'}(F_2, {}^i\lambda_{2n})]$.

Furthermore, let the product $C(i, \rho, n, k) = [V_{1j}(F_1, {}^i\lambda_{1n}) \cdot V_{2j'}(F_2, {}^\rho\lambda_{1k}(F_1))]$, $k < n$, represent a coincidence produced by a directly transmitted state of a subsystem S_{1n} incident on the field F_1, belonging to a generation n, and a state of a subsystem S_{1k} first reflected by F_1 and then incident on the field F_2 and transmitted inside it, hence belonging to a preceding generation k. The difference $n-k$ is here regarded as a measure of the time necessary for a subsystem S_1 ro reach first F_1 by direct incidence, and then F_2, after one reflection on F_1. The first value V_{1j} from $C(i, \rho, n, k)$ is informed of the characteristics of F_1 by interaction with which it emerges, but—according to (10)—it is not informed on F_2. As to the second value $V_{2j'}$, it is now informed *both* of F_1 (via the reflected state ${}^\rho\lambda_{1k}$ which bears the marks of characters of F_1 acquired during the process of reflection on F_1), and of F_2, by interaction with which it emerges.

Let the product $C(\rho, i, k, n) = V_{1j}(F_1, {}^\rho\lambda_{2k}(F_2))\, V_{2j'}(F_2, {}^i\lambda_{2n})$ denote a coincidence produced by a state of a subsystem S_{2k} reflected by F_2 and then incident on F_1 and transmitted inside F_1, belonging to a generation $k<n$, and a state of a subsystem S_{2n} directly incident on F_2 and transmitted inside F_1. *Mutatis mutandis*, the comments on dependences are the same as for $C(i, \rho, n, k)$.

Finally, let the product $C(\rho, \rho, n, k) = V_{1j}(F_1, {}^{\rho}\lambda_{2n}(F_2)) \cdot V_{2j'}(F_2, {}^{\rho}\lambda_{1k}(F_1))$ denote a coincidence produced by a state of a subsystem S_{2n} reflected by F_2 and then incident on F_1 and transmitted inside F_1, belonging to a generation n, and a state of a subsystem S_{1k}, $k \leqslant n$, reflected by F_1 and then incident on F_2 and transmitted inside F_2. In a coincidence $C(\rho, \rho, n, k)$ each of the two registered values is informed of both Stern–Gerlach fields.

The preceding classification of the coincidences entails that, for each field, *two* distinct directions of incidence do realize: a direction of direct incidence which depends only on the orientation of that field, and a direction of incidence after reflection on the other field, which, for any values R_1 and R_2, encodes the—invariant—angle between the orientations of the two fields.

In consequence of the hypotheses H_1 and H_3 each one of the four types of coincidences defined is separable by construction. So *Einstein separability is ensured, notwithstanding the fact that the values registered via the devices D_{1A} are admitted to depend in general on the settings from D_{2A}, and vice versa.*

Representation. Let us now define the statistical representation corresponding to the achieved classlification of the coincidences.

The four classes of possible coincidences defined above constitute disjoint events. These put a partition on the universe of coincidences. So the total probability of a coincidence is the sum of the probabilities of the classes. Then the expectation value of the products $V_{1j} \cdot V_{2j'}$ brought forth by the coincidences (which identifies here with the correlation function) is a sum of terms.

Now, the quantum mechanical prediction $\langle \boldsymbol{\sigma}_1 \cdot \mathbf{u}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{u}_2 \rangle = -\mathbf{u}_1 \cdot \mathbf{u}_2$ (defined before) presupposes restriction to coincidences which bring in, each, only *one* pair, i.e., restriction to coincidences $C(i, i, n, n)$ and $C(\rho, \rho, k, n)$ with $k = n$. So the representation to be compared with the theoretical quantum mechanical prediction is the sum

$$\begin{aligned} \exp(V_{1j} V_{2j'}) = & \iint V_{1j}(F_1, {}^{i}\lambda_{1n})\, V_{2j'}(F_2, {}^{i}\lambda_{2n})\, \pi({}^{i}\lambda_{1n}, {}^{i}\lambda_{2n}\, d({}^{i}\lambda_{1n})\, d({}^{i}\lambda_{2n}) \\ & + \iint V_{1j}(F_1, {}^{\rho}\lambda_{2n}(F_2))\, V_{2j'}(F_2, ({}^{\rho}\lambda_{1n}(F_1)) \\ & \times \pi({}^{\rho}\lambda_{2n}(F_2), {}^{\rho}\lambda_{1n}(F_1))\, d({}^{\rho}\lambda_{2n}(F_2))\, d({}^{\rho}\lambda_{1n}(F_1)) \end{aligned} \tag{11}$$

with the condition on norm

$$\Sigma_C \pi(C) = \pi(C(i, i, n, n)) + \pi(C(\rho, \rho, n, n)) = 1$$

where $\pi(C)$ is the total probability of a coincidence of type C, integrated with respect to the λ-variables on which that type depends.

But it is important to be aware of the fact that, as soon as the intensity of the source of pairs is not low enough for ensuring experimentally the restriction to coincidences $C(i, i, n, n)$ and $C(\rho, \rho, n, n)$, the *measured* mean corresponds to the more general representation

$$\begin{aligned}
\exp(V_{1j}V_{2j'}) = &\iint V_{1j}(F_1, {}^i\lambda_{1n})\, V_{2j'}(F_2, {}^i\lambda_{2n})\, \pi({}^i\lambda_{1n}, {}^i\lambda_{2n})\, d({}^i\lambda_{1n})\, d({}^i\lambda_{2n}) \\
&+ \iint V_{1j}(F_1, {}^i\lambda_{1n})\, V_{2j'}(F_2, {}^\rho\lambda_{1k}(F_1)) \\
&\times \pi({}^i\lambda_{1n}, {}^\rho\lambda_{1k}(F_1))\, d({}^i\lambda_{1n})\, d({}^\rho\lambda_{1k}(F_1)) \\
&+ \iint V_{1j}(F_1, {}^\rho\lambda_{2k}(F_2))\, V_{2j'}(F_2, {}^i\lambda_{2n}) \\
&\times \pi({}^\rho\lambda_{2k}(F_2), {}^i\lambda_{2n})\, d({}^\rho\lambda_{2k}(F_2))\, d({}^i\lambda_{2n}) \\
&+ \iint V_{1j}(F_1, {}^\rho\lambda_{2n}(F_2))\, V_{2j'}(F_2, ({}^\rho\lambda_{1k}(F_1)) \\
&\times \pi({}^\rho\lambda_{2n}(F_2), {}^\rho\lambda_{1k}(F_1))\, d({}^\rho\lambda_{2n}(F_2))\, d({}^\rho\lambda_{1k}(F_1)) \qquad (12)
\end{aligned}$$

with the condition on norm

$$\begin{aligned}
\Sigma_C \pi(C) = &\pi(C(i, i, n, n)) + \pi(C(i, \rho, n, k)) + \pi(C(\rho, i, k, n)) \\
&+ \pi(C(\rho, \rho, k, n)) = 1
\end{aligned}$$

The characteristics of the representation (11), by themselves, entail already a structure of important conclusions. Indeed:

The representation (11) is Einstein-separable, in consequence of the individual Einstein-separability of each one of the two constituting coincidences. It is even trivially Einstein-separable: *It could be simulated by the help of an ensemble of Newtonian balls.* (This semantically inacceptable quasi-classical character of the representation (11) is entailed by the over-simplifying hypotheses H_2 and H_3.)

The representation (11) is fundamentally different from Bell's. The difference stems from Bell's assumption[1] that "The result A of measuring $\boldsymbol{\sigma}_1 \cdot \mathbf{a}$ is then determined by $\mathbf{a}$ and λ, and the result B of measuring $\boldsymbol{\sigma}_2 \cdot \mathbf{b}$ in the same instance is determined by **b** and λ.... The vital assumption[2] is that the result B for particle 2 does not depend on the setting **a** of the magnet for particle 1, nor A on **b**." (The correspondences with our notations are obvious. Bell's reference 2 sends to Einstein's principle of separability.) The treatment which led to (11) shows by construction that nothing necessarily entails the quoted restriction, neither the fact that a hidden variable approach is attempted, nor Einstein's principle of separability, nor quantum mechanics. The result A can, in general, depend on **b**, Einstein-separably, and B on **a**. This is so because—in consequence of the existence of reflection—the result B might not be for "a particle 2," the result A might not be for "a particle 1": *the particles can be interchanged.* Furthermore, the representation (11) cannot be absorbed in Bell's by some generalization, while the inverse is true: Bell's representation is reobtained from (11)—formally—by setting $R_1 = R_2 = 0$.

So Bell's Einstein-separable representation is not the most general one conceivable.

Though it is Einstein-separable, the representation (11) is Bell-nonlocal, since it incorporates crossed dependences, while from Bell's representation any crossed dependence is banished via the "locality" condition. So the representation (11) shows by construction that

Einstein-separability and Bell-locality are distinct qualifications. Bell nonlocality does not necessarily entail nonseparability in Einstein's sense. Bell-locality is not a necessary condition for Einstein-separability, though it might be a sufficient condition.

The enumerated characteristics of the representation (11) indicate already rather clearly that the usual interpretation of Bell's theorem, namely as a proof of incompatibility between quantum mechanics and relativity, cannot be accurate. Nevertheles we shall now prove explicitly that the representation (11) entails a counterexample to this interpretation.

Counterexample. The expectation value (11) should always equal the quantum mechanical expectation. Is this possible? With Bell's Einstein-separable representation, it is not. But we assert that

With the Einstein-separable and Bell-nonlocal representation (11), *Bell's theorem cannot be obtained any more.*

Proof. Consider first the general case $R_1 \neq 0$ or $R_2 \neq 0$. In this case, according to the significance assigned to the hypothesis H_2, the representation (11) contains contributions from reflected states ${}^{\rho}\lambda$. Then (11) differs from Bell's representation and the inequality cannot be proved, namely because *Bell's groupings by common factors*[1] *cease to be possible*, in consequence of the existence of crossed dependences posed in the arguments of the factors.

But the framework (11) permits theoretically also the particular case $R_1 = R_2 = 0$. In this case, according to the hypothesis H_2, no reflected states ${}^{\rho}\lambda$ exist. So the second term of (11) vanishes and (11) reduces to its first term normed to 1. This—up to notations—is precisely Bell's representation. Now, in Bell's proof, this representation is asserted for any angle θ. However, the proof makes use of only three distinct angles θ (which are combined in three distinct pairs of angles). So if the hypothesis $R_1 = R_2 = 0$ were possible for at least three angles θ (and with values such that they do produce the inequality) this would entail resurrection of Bell's proof starting from the Bell-nonlocal representation (11). However, according to Newtonian quantum mechanics, the mentioned condition cannot be strictly realized. The physical realization of the condition $R_1 = R_2 = 0$ could be envisaged, for a given value of θ and for one Stern–Gerlach field denoted F_q, $q = 1, 2$, only via the realization of total transmission through the potential corresponding to this field, i.e., via the realization of the condition $k_q l_q = n\pi$ (k_q is the wave vector of S_q, n is an integer, l_q is the depth of the field, and π measures here a half wavelength).[21] But the wave function of the microsystem S_q contains always more than only one Fourier component, so it involves more than only one value k_q. Moreover, not only one direction of incidence on F_q is involved, but all the directions of incidence corresponding to the three angles θ considered in the proof. Even though the integer n is not fixed, one given inhomogeneous Stern–Gerlach field cannot be structured so as to ensure, rigorously, $k_q l_q = n\pi$ for *all* these directions of incidence *and* for all the wave-vector values k_q involved. Hence the condition $R_1 = R_2 = 0$ cannot be strictly fulfilled.

Bell's proof does not establish *that* any *Einstein-separable representation is incompatible with quantum mechanics.*

Then what does the proof establish?

Bell's proof establishes only that inexistence of crossed dependences, i.e., Bell-"locality," is incompatible with quantum mechanics.

So

Existence of crossed dependences, i.e., Bell-"non-locality," has to be radically distinguished from violation of Einstein's principle of separability.

These conclusions are a *syntactic fact.*

But now, what about the *semantic* implications of the relation between the representation (11) and Bell's theorem?

Critique of (11). To begin with, any physicist who works with quantum mechanics knows that the representation (11) cannot be factually true: The microsystems cannot be imagined like Newtonian balls, since their behavior calls in irrepressibly the notion of corpuscular waves also. The representation (11) is not more than a useful conceptual analyzer.

Furthermore, the representation (11) and its consequences are very sensitive to the numerical values of the reflection coefficients R_1 and R_2, while this sensitiveness seems to be absent both in the quantum mechanical prediction and in the experimental facts: In Aspect's experiments, where photons and polarizers were used instead of massive particles and fields, ideally, the reflection coefficients introduced by the polarizers can be considered to be null, and nevertheless the quantum mechanical correlations did emerge.

Finally, the preceding counterexample-proof has a limiting character: Even if any wave packet describing a realizable state contains more than only one Fourier component, this limit can be approached as closely as one wants. Furthermore, for one angle θ and one wave-vector value k_q, the condition $k_q l_q = n\pi$ might, perhaps, be somehow realizable, namely by choosing the depths l_{qa} and l_{qb} of the fields F_1 and F_2 along the two directions corresponding to θ, such as to have $k_q l_{qa} = n_a$, $k_q l_{qb} = n_b$, n_a and n_b being two distinct integers. This could perhaps be realizable even for several discrete directions. Now, if all this were possible indeed, the limit $R_1 = R_2 = 0$, crucial for the proof of the counterexample, might be approachable as closely as one desires. Then the resurrection of Bell's proof, starting from (11), could be approached as closely as desired. This possibility renders unstable the status of our counterexample.

So, within the framework created by the concept of reflected dependence, the representation (11) and Bell's representation make contact, formally, via the unique "point" $R_1 = R_2 = 0$. While according to the counterexample-proof this formally existing extremity is not strictly realizable, which has a decisive role, Bell's proof, on the contrary, is entirely located on precisely this point, as if the nullity of the quantum mechanical reflection coefficients R_1 and R_2 were simply devoid of any importance.

Under these conditions, what, exactly, can be concluded—from a semantic point of view—from the preceding counterexample to the current interpretation of Bell's theorem?

Suggestion. When it is examined inside the particular framework defined here, Bell's proof suggests that the implicit syntaxis of quantum mechanics might assert crossed dependences even *if* the condition $R_1 = R_2 = 0$ were *strictly* fulfilled. And Aspect's results suggest that *at the photonic limit, with ideal polarizers, this situation might factually arise.*

So our conclusion concerning the semantic implications of the relation between the representation (11) and Bell's proof is as follows.

The representation (11) captures a fundamental semantic implication of the quantum mechanical formalism, which seems not to have been noticed so far, namely *the connection between the quantum mechanical* *crossed dependences and reflection.* But the hypotheses contained in the representation (11) are amputing, and so *the representation* (11) *is factually false.* It fails to incorporate another fundamental semantic implication of quantum mechanics, namely the existence of contributions to the quantum mechanical crossed dependences, also from another source than that indicated by nonnull quantum mechanical reflection coefficients—probably from some formal correspondent of the existence also of reflections of pure-wave aspects of the microsystems.

The remainder of this work is devoted to a more complete understanding of the quantum mechanical crossed dependences and of the relations between these and separability in Einstein's sense. This improved understanding will lead to a factually acceptable hidden representation of the quantum mechanical mean, to be substituted to the factually unacceptable representation (11).

5. DE BROGLIE'S MODEL, RELATIVITY, AND QUANTUM MECHANICS

Questioning the hypotheses admitted in the representation (11) and their relation to Einstein separability amounts to a more profound investigation concerning the model assignable to a microsystem. This brings us back to Louis de Broglie's works.

Relativity versus $p = h/\lambda$. In the first five pages (19–24) of his thesis Louis de Broglie[24] pictured a heavy microsystem as an entity consisting of

(a) a strongly localized fragment of energy $W = mc^2$, labelled by the corresponding mass $m = m_0/\sqrt{1 - v^2/c^2}$ (v is the velocity of the position of the mass m, with respect to the observer who assigns the mass value m; c is the velocity of light; m_0 is the value of m as assigned by an observer with velocity $v = 0$ relative to this mass) and endowed with an *internal, clocklike* frequency ν_c;

(b) an extended physical wave surrounding the fragment of energy $W = mc^2$ and characterized by a *wave* frequency with value

$$\nu_\omega = mc^2/h = W/h \tag{12}$$

for coherence with the Einstein–Planck relation between wave frequency and energy.

The frequencies ν_c and ν_ω are distinguished from one another by their relativistic variance: When $v=0$, ν_c and ν_ω are numerically equal,

$$v=0 \rightarrow \nu_{0c}=\nu_{0\omega}=m_0 c^2/h \tag{13}$$

But when $v \neq 0$, ν_c becomes

$$\nu_c=\nu_{0c}\sqrt{1-v^2/c^2} \tag{14}$$

which is the Einstein variance for *clock* frequencies, while ν_ω is posed by de Broglie to become

$$\nu_\omega=\nu_{0\omega}/\sqrt{1-v^2/c^2} \tag{15}$$

in agreement with its definition (12), which is *another* variance, also relativistic, since it is entailed by the relativistic variance of the mass, but concerning specifically the *corpuscular—wave* frequencies. So ν_c and ν_ω label two phenomena of essentially different nature. However, in de Broglie's model these phenomena are organically tied to one another. Indeed, de Broglie has stressed that "one" microsystem can possess *stably* the described structure only if the two sorts of periodicities labelled by ν_c and ν_ω remain constantly in phase on the whole frontier between the localized fragment of energy $W=mc^2$ and the wave which surrounds it. Furthermore, he has proved a theorem according to which, in the absence of fields, this "harmony of phases" can subsist if and only if the velocity v of the localized mass m and the phase velocity V of the wave associated with it are connected by the relation

$$vV=c^2 \tag{16}$$

So $v<c$ *entails* $V>c$: In the void, the phase of de Broglie's "corpuscular" wave has supraluminal velocity.

Futhermore, in de Broglie's approach the relations (12)–(16) are equivalent to the famous relation $p=h/\lambda$[25]. Indeed:

For the observer at rest with respect to the mass-energy frament, who measures the value m_0, the surrounding wave is describable by the expression

$$\Psi_0(x_0, t_0)=a\exp(2\pi i\nu_{0\omega}t_0)$$

where a is a constant amplitude and the phase $2\pi\nu_{0\omega}t_0$ is independent of the space coordinates. For the observer who moves with velocity v with respect to the mass-energy fragment and measures the value m, the wavelike phenomenon which surrounds the mass is posed to possess the form of a plane progressive wave of which the structure is obtained from that, $a\exp(2\pi i\nu_{0\omega}t_0)$, perceived by the observer at rest with respect to the mass, via the Einstein-Lorentz transformation for the time coordinate, $t_0=(t-\beta x/c)/\sqrt{1-\beta^2}$. So

$$\Psi(x, t)=a\exp(2\pi i\nu_\omega(t-x/V))=a\exp(2\pi i\nu_{0\omega}(t-\beta x/c)/\sqrt{1-\beta^2}))$$

The last equality can hold only if $\nu_\omega=\nu_{0\omega}/\sqrt{1-\beta^2}$, $V=c/\beta=c^2$, i.e., $vV=c^2$. Now, comparing the phases of Ψ_0 and of Ψ, and taking into account that $W_0=m_0c^2=h\nu_{0\omega}$, $W=mc^2=h\nu_\omega$, $p=Wv/c^2$, one finds for the coefficient of x the condition $\nu_\omega/V=(W/h)\times(\beta/c)=(1/h)\times(Wv/c^2)=p/h$.

The relation $p=h/\lambda$, which lies at the basis of quantum mechanics, is the direct consequence of relativistic requirements, and these lead—jointly—to the assertion of infraluminal values $v<c$ for the velocity v of the mass-energy fragment from a de Broglie microsystem, and of supraluminal values $V>c$ of the phase velocity of the corpuscular wave of this microsystem.

Relativity versus Self-Microfields. After his thesis, Louis de Broglie developed his model, adding essential features.[25]

In the first place, the phase of the (three-dimensional) wave function $\mathbf{\Psi}(\mathbf{r}, t)=a(\mathbf{r}, t)\,e^{i\varphi(\mathbf{r},t)}$ (standard notations) describing the wave of the microsystem is posed to be tied to the velocity of the fragment of localized mass-energy of the microsystem, via the "guidance" relation

$$\mathbf{v}=-c^2\,\mathbf{grad}\,\varphi/(\partial\varphi/\partial t) \tag{17}$$

where

$$[(\partial\varphi/\partial t)=W=h\nu]=M_0c^2/\sqrt{1-v^2/c^2} \tag{18}$$

with

$$M_0=\sqrt{m_0^2+h^2/4\pi^2(\Box a/a)} \tag{19}$$

designating a mass which is variable as soon as the d'Alembertian $\Box a$ of the amplitude a of the wave is nonnull. Now, according to de Broglie's description, this d'Alembertian is nonnull as soon as the wave of the considered microsystem superposes on itself, autointerferes, thereby producing a self-micropotential. From this derive self-microfields which act on the velocity $\mathbf{v}$ of the mass-fragment of the microsystem, creating a self-dynamics. In particular, *when a microsystem encounters an obstacle its wave might be partly reflected even in the particular cases when its piece of mass-energy is transmitted.* The wave might even be systematically split in a transmitted part and a reflected part, only one of which carries with it the piece of mass-energy of the microsystem. In this case an *observable* interference "state" emerges.[25]

The new specifications (17), (18), (19) assign very peculiar specificities to the interference states. These are so suggestive that it seems worthwhile to reproduce here the essence of an illustration.[25]

Consider an imperfectly reflecting mirror with reflection coefficient $0<\eta<1$. A microsystem S with state vector $|\Psi_1\rangle$ is incident on the mirror. Inside the domain of interference the state has an interference form $|\Psi_{12}\rangle = a|\Psi_1\rangle + b|\Psi_2\rangle = |ae^{i\varphi}\rangle$ with amplitude

$$a^2 = \text{const}[1+\eta^2+2\eta(\cos 4\pi(v/V)z\cos\theta+\delta)] \tag{20}$$

The velocities v and V as defined by (17) are

$$\begin{aligned} v_x &= -c^2(\partial\varphi/\partial x)/(\partial\varphi/\partial t) = c^2/V(\sin\theta) = v_0\sin\theta \\ v_y &= 0 \\ v_z &= -c^2(\partial\varphi/\partial z)/(\partial\varphi/\partial t) = c^2/V[\cos\theta(1-\eta^2)/a^2] = v_0\cos\theta[1-\eta^2)/a^2] \end{aligned} \tag{21}$$

So

$$v^2 = v_x^2 + v_z^2 = c^4/V^2[\sin^2\theta + \cos^2\theta((1-\eta^2)^2/a^4)] \tag{22}$$

Now, for values of z such that the cosinus from a^2 is -1 the fringes are obscure, while for values of z which make the cosinus 1 the fringes are brilliant. So for the obscure fringes

$$a^2 = (1-\eta)^2, \qquad v^2 = c^4/V^2[\sin^2\theta + \cos^2\theta((1+\eta^2)/(1-\eta)^2)] \tag{23}$$

This value of v^2 can become superior to c^2 (if c^4/V^2 and if $(1+\eta)/(1-\eta)$, always superior to 1, is big enough).

So in an interference state the relations $vV=c^2$, $v<v$, $V>c$, cease to hold in general, and supraluminal values can appear also for the velocity of the mass-energy fragment $W=mc^2$ contained by a de Broglie microsystem, not only for the phase velocity of its wave. A more detailed analysis shows that such supraluminal values of the velocity v, if they do appear, appear periodically (while the obscure fringes are traversed), and while they are realized the variable mass M_0 from (19) becomes imaginary: a way of expressing formally a sort of periodic dissolution and recondensation of the variable mass M_0.

Such phenomena—even though they are organically connected with the relativistic method of thinking and of describing reality—fall nevertheless entirely outside the domain of application of what is nowadays called "the theory of relativity." For instance, the very concept of a continuously existing *moving object with a permanently defined mass and velocity loses its significance inside a de Broglie microscopic interference field.*

The Pathologic Growth of Quantum Mechanics. The model defined by de Broglie had not the time to impose itself. While it was just beginning to be born—rooted in the relativistic thinking—a conceptual skid tore it off and displaced on another ground the developments initiated by it. All this happened in the first seven pages of Louis de Broglie's Thesis.

In the first five pages de Broglie sketched out his model. This model is essentially *individual* and so it has been possible to connect it directly with Einstein's relativity—which basically is individual—namely by the specification of the two different relativistic variances assigned: one, to the localized clocklike frequency ν_c, and the other one, to the wave frequency ν_ω [see (12) and (13)]. But on the sixth and seventh page (labelled 25 and 26 in the quoted edition) a surreptitious intellectual accident happened, a sort of loss of control. Urged by a desire to produce a formal representation of the spatial location of the mass-energy fragment m, and to specify the value of its velocity v, de Broglie makes the following reasoning:

"Si l'on attribue au mobile une vitesse $v=\beta c$ en ne donnant pas à β une valeur tout à fait déterminée, mais en lui imposant seulement d'être comprise entre β et $\beta=\delta\beta$, les fréquences des ondes correspondantes remplissent un petit intervalle ν_ω, $\nu_\omega+\delta\nu_\omega$. Nous allons établir le théorème suivant qui nous sera très utile ultérieurement. 'La vitesse du groupe des ondes de phase est égale à la vitesse du mobile.'" (Our labelling by ω, for clarity.)

Then he writes $\beta=v/c$, $V=c/\beta$, $\nu_\omega=(1/h)(m_0c^2/\sqrt{1-\beta^2})$, $1/U=d(\nu_\omega/V)/d\nu_\omega$, $U=(d\nu_\omega/d\beta)/d(\nu_\omega/V)/d\beta$, where U is the group velocity. From this he draws $U=\beta c=v$. He concludes:

"La vitesse de groupe des ondes de phase est bien égale à la vitesse du mobile."

These formulations are flawed by dissolving ambiguities.

Indeed, in the initial model, one microsystem contains one "mobile" (consisting of a piece of mass-energy $W=mc^2$) possessing one velocity v, and one wave possessing one phase velocity V, these two sorts of velocities being tied to each other by a one-to-one relation $vV=c^2$. Everything was strictly individual and uniquely defined. While since $\beta=v/c$, $\delta\beta=(1/c)\,\delta v$. So, when a small interval of wave frequencies ν_ω, $\nu_\omega+\delta\nu_\omega$ is considered and—in connection with this, via $\nu_\omega=1/h(m_0c^2/\sqrt{1-\beta^2})$—*derivations are performed on* β, also a small *interval* of mass velocities, $v, v+\delta v$, is introduced. Thus a whole *ensemble* of complete de Broglie *microsystems* is now associated with the unique microsystem considered initially, not only an ensemble of waves considered in connection with a unique microsystem. This ensemble, quite remarkably, plays indeed the role of an element of reference permitting a mathematically formulated labelling of the position of the mass m (the position of the maximum of the amplitude of the packet of waves), and it yields indeed a mathematical definition of the value of the corresponding velocity v. While in the initial model, m and v are only symbolized and stated to exist and to possess a value, but these values are not represented and calculated. On the other hand, the mentioned ensemble of microsystems is introduced inexplicitly and without ensuring a sharp distinction with respect to the *one* microsystem which is *studied*: There is no indication as to *which* is "the"—one—velocity v "but without giving to β an entirely determined value" (notice that $\beta = v/c$, with c constant!). In consequence of this the one studied microsystem loses its identity. The ensemble of microsystems, which in fact is introduced, covers the studied microsystem and blurs out the one-to-one correspondences $v_\omega \Leftrightarrow v$, $v \Leftrightarrow v_c$, and $v \Leftrightarrow V$ from the individual model. So these ceased to retain attention. And so the ensemble of microsystems involved in the Broglie's theorem on the group velocity dissolved the individual model. This model had no time to penetrate into the physicist's minds. The ensemble shifted the description too rapidly on the statistical level.

As to this ensemble of microsystems which replaced the initial model, the implicit character of its introduction entailed a ghostification of its "corpuscular" contents. Only the explicitly mentioned wave aspects subsisted. In this way the initial de Broglie microsystem transmuted in "a wave packet capable of corpuscular manifestations." The era of the wave-particle "complementarity" began.

When a statistics is built by explicit supression of explicitly posed individual qualifications, the probabilities defined by it are clearly due to "ignorance." But if no reference to an individual model is available, these probabilities are perceived as "essential." The era of the problem of completeness began.

The concept of a wave packet introduced the linear Fourier calculi. To form a packet, integrations over $p = mv = h/\lambda$ from $-\infty$ *to* $+\infty$ were performed, *without specifying any more whether* $|v|$ *was or was not limited by* c. The Fourier calculi were then extended into a generalized spectral formulation, associated with the linear formalism of kets, bras, and linear operators, and so the traces of the initial rooting in relativity entirely disappeared. *The era of the search for a connection between relativity and modern microphysics began nearly as soon as modern microphysics had been generated by relativistic thinking.*

A rumor of philosophic questionings accompanied the process, like a greek chorus. The positivistic rules and interdictions were erected as a dam against this rumor. From time to time, in the strong mind of a Schrödinger, an Einstein, a Wigner, a Bell, the questionings rose to an intensity and a degree of organization sufficient for the condensation of a lasting "paradox." These paradoxes point toward the hidden source of the persisting interrogations, namely *the ambiguities involved in the representation of one—complete—microsystem, by a packet of—only—waves.*

A botched concept lies at the basis of quantum mechanics. While repelled beneath the formalization, de Broglie's unexplored model cryptically signalizes its presence and its necessity.

The concept of a microsystem will have to be rethought. A deep unification between quantum mechanics and relativity will be obtained only if the relativistic significances captured in the initial relations $\nu_{\omega}=\nu_{0c}/\sqrt{1-\beta 2}$, $vV=c^2$, are explicitly integrated in a further formalization.

De Broglie–Bohr Nonseparability vs. Einstein Nonseparability. The linear formalism of Newtonian quantum mechanics has incorporated certain wave aspects of de Broglie's initial relativistic model. In particular, it has incorporated supraluminal phase velocities. For instance, any eigenket of the momentum operator is a de Broglie wave of which the phase velocity tends toward ∞ when $p=m_0v$ tends towards 0. And when, in the presence of steep potentials, the linear formalism introduces stationary solutions of the equation of evolution, consisting of interferent superpositions of incident and reflected terms, this formalism asserts implicitly reflected dependences instaured—at least formally— via supraluminal propagations of phases of plane components from the packets which are used.

This is the de Broglie–Bohr nonseparability of quantum mechanics, directly inherited from de Broglie's model.

This sort of nonseparability is an essential character of the theory. Negation of this character would amount to a global negation of quantum mechanics.

Now, this de Broglie–Bohr "quantum nonseparability" is not in contradiction with relativity. Quite the contrary, it is rooted in relativity via the basic relation $vV=c^2$.

So the de Broglie–Bohr quantum nonseparability probably somehow confines Einstein's notion of separability. Thereby it probably also completes it, inside some unified, general qualification of the relations between waves and energy fragments, and of the transition from nonheavy energy fragments to heavy energy fragments. Like the principle of correspondence, this supposed delimination might be significant also for the description of the transition from microscopic, individual characterizations, to macroscopic, statistical ones.

The belief in the necessity of a radical choice to be made between quantum nonseparability and Einstein separability probably constitutes a misleading flattening of the conceptual volumes involved, a sort of projective amputation of a still unknown conceptual structure, while a sharp mutual individualization of these two separability qualifications could be the germ of a veritable fusion between relativistic field theories and relativistic descriptions of heavy systems and, hence, between microphysics and relativity.

6. AN IMPROVED REPRESENTATION

Louis de Broglie has not succeeded in building a satisfactory alternative to quantum mechanics.[26] While his genious is manifest in his intuitive graspings of the profound texture of what is named "reality," he did not possess a high sense of logical structure. Nevertheless, the de Broglie model has founded the present quantum mechanics. And now it suggests very important qualitative ideas which, in particular, define the features of the causal representation which will be studied below.

Reflection of Pieces of Mass-Energy and Reflection of de Broglie Waves. The quantum mechanical coefficients of reflection and of transmission concern exclusively "corpuscular" aspects, able to produce observable localized effects on a sensitive screen, on a counter, etc. So, in terms of the de Broglie model, the coefficients R_1 and R_2, T_1 and T_2, count relative frequencies of individual reflections or of individual transmissions *of the piece of mass-energy contained by one microsystem*, when the chain [preparation–evolution–measurement] is reiterated for that microsystem a great number of times. As to the hypothetical de Broglie reflections of exclusively a wave part of a microsystem, *they are devoid of any explicit representation inside the quantum mechanical formalism.* But implicitly, the quantum mechanical description might somehow incrypt them in the ket obtained as solution of the evolution equation of the problem.

How?

The answer can be suggested in two steps.

Macroscopic Exterior Fields and Microscopic Self-Fields. In the quantum mechanical formalism only the macroscopic constraints which act on the studied microsystem are represented formally. The formal representation of these macroscopic constraints is achieved by the potential term from the Hamiltonian of the corresponding Schrödinger equation, and by the limiting conditions—involving the various obstacles—which select the solution. As to microscopic characterizations of the studied system, they are all only implicitly contained in the solution, a normed ket $|\Psi(\mathbf{r}, t)\rangle$ representing the studied microsystem's state.

This implicit content remains cryptic. It accedes to cognoscibility only in terms of results of measurements of quantum mechanical operators. This—as we have shown in detail in Chapter 2—is an access both delayed with respect to the time label from the argument of the ket, and obtained in the form of ensembles of events which are located on a dust of disjoint space-time supports. These emerge successively by reiterations of the chain [preparation–evolution–measurement], and they intersect the space-time supports of the measuring apparatuses, i.e., they do not belong exclusively to the microsystem itself: a fragmentation, adulteration, and coded transposition of a hypothetical intrinsic form which is strictly undescribed by the quantum mechanical formalism. A form hardly supposed to occupy the space-time domain labelled $(\mathbf{r}, t)$ in the argument of the ket associated to the studied microsystem.

According to de Broglie's model, this unknown form involves a "corpuscular" wave which is affected (distorted, or split, or reflected, or diffracted) by the macroobjects and by the macrofields with which it interacts. In consequence of its interactions with macroscopic fields and objects the microsystem's corpuscular wave can superpose and interact with itself, or it can interfere with itself, thereby generating self-microfields of the microsystem ($\Box a \neq 0$ in the expression (19) of M_0). These act on the value and on the velocity of the variable mass M_0 possessed by the microsystem [see (17)–(19)]. Such processes happen systematically in any interference situation, and interference situations emerge—locally and transitorily—as son as the evolution of the studied microsystem is not free, not free of fields, or not free of obstacles, or of both.

The interference situations are the rule for a microsystem. The exception consists of the noninterference situations. What is usually qualified as an interference "state" is in fact only a very particular type of interference, characterized by stability and by a macroscopic spatial extension, ensuring observability of specific effects. The specific *observable* characters of these particular, extended and stable interference situations constitute a transposition of the de Broglie intrinsic description, a transposition commanded by the operational-observational code $(1) \rightarrow (8)$ of quantum mechanics. This code dislocates and breaks up the intrinsic description. So the existence of the microscopic self-fields, and the processes induced by them, can be only guessed behind the quantum mechanical formalism, indicated to the forewarned believer by dimmed signs.

So during the Schrödinger evolution of a microsystem, probably certain microphenomena occur quite currently—generated by microscopic self-fields—which, while they transgress the theory of relaltivity as it now stands, are also devoid of any explicit representation inside the present quantum mechanics.

This probably holds, in particular, for the interference field which arises during the process of emergence of an individual result of a spin measurement on pairs.

Let us now continue by asking an important question.

The Whole Packet, or Only One de Broglie Component? In an individual result produced by a spin measurement on pairs, does the whole quantum mechanical packet Ψ act, or only one de Broglie component? We have shown in what sense a wave packet corresponds to a whole ensemble of de Broglie microsystems, each one of which involves one corpuscular wave.

The de Broglie model and the genesis of the concept of a wave packet suggest that in the process of emergence of one observed result produced by a measurement, not the whole quantum mechanical wave-packet is concerned, but only one of its de Broglie components.

Now, the phase velocity of a de Broglie wave is always supraluminal (which is not true concerning the phase velocity of the group). So the reflected part of the de Broglie wave of the subsystem S_{1n}—whether its "corpuscular" piece of mass-energy is transmitted or whether it is equally reflected—can reach the domain where evolves the subsystem S_{2n} belonging to the *same* generation, *before* the corpuscular part of S_{2n} arrives on the field F_1. So the reflected part of the wave of S_{1n}, which bears information concerning F_1, interacts with the subsystem S_{2n} (it interferes with its state—in the full sense of the term, since S_{1n} and S_{2n} are "coherent" in consequence of their common birth) *before* S_{2n} interacts with F_2 and eventually is transmitted, thus producing an observable result. *This interference might imprint on the state "of S_{2n}"—now, in fact, mixed with a part of the state of S_{1n}—information on F_1, and this information might act on the emergence of the observable result.*

We also notice that, before the emergence of an observable result, the interference field, once created, might generate microscopic processes which transgress the relativistic descriptions achieved in the theory of relativity as it now stands. In particular, they might produce periodic dissolutions and reconcentrations of the variable mass M_0 of the subsystem S_{2n}, resulting globally in a supraluminal translocation of this mass.

The Photonic Limit: Heteromicrointerference. For photons the localized energy bundle $h\nu$ has the limiting velocity $v = c$. So, $vV = c^2$, the phase velocity V is equally c. *So a reflected corpuscular wave can superpose only on a bundle $h\nu$ from a* preceding *generation.* Now, the specificities of the Bose–Einstein statistics and those of laser phenomena suggest that, notwithstanding this heterogeneity of birth, the mentioned superposition could entail interaction, a sort of heteromicrointerference; i.e., it could influence the behavior of the energy-bundle $h\nu$. This is no more than a conjecture. But this conjecture might have a vital importance for the interpretation of Aspect's experiments.

The Final Representation. The preceding conjectures can now be used for a redefinition of the content assigned to the possible classes of coincidences distinguished in Sec. 4.

We include from now on the photonic limit. In order to stress this, the symbols F_1 and F_2 used until now for indicating Stern–Gerlach fields will from now on be replaced, respectively, by the symbols P_1 and P_2, indicating polarizers when photons are concerned, or fields when massive particles are concerned.

Furthermore, we modify the designata of the symbols introduced in Sec. 4 and we add new symbols, in such a way as to be able to express de Broglie's model and, in particular, to distinguish explicitly between a part of the wave of a microsystem which does carry a mass-energy fragment, and a part which does not.

The symbols S label a microsystem globally, with its mass-energy fragment and its wave. So a pair can be symbolized by (S_q, $q = 1, 2$), where $q = 1$ and $q = 2$ mean, respectively, emitted toward P_1 and emitted toward P_2.

Any parameter λ labels a part of the wave of a microsystem. If it labels a part which carries a mass-energy fragment, it is distinguished by a left upper-index c (corpuscle, or clock): ${}^c\lambda$. Thereby each one of the two categories of symbols λ—incident, ${}^i\lambda$, and reflected, ${}^\rho\lambda$—is split now into two new categories, each one qualified furthermore by the index q of direction of emission, and by an index of generation g:

${}^{ci}\lambda_{qg}$ and ${}^{c\rho}\lambda_{qg}$ designate parts of a microsystem's wave which carry a localized fragment of energy, while ${}^i\lambda_{qg}$ and ${}^\rho\lambda_{qg}$ designate pure wave parts of a microsystem's wave (g: generation of the part; $q = 1, 2$: direction of emission).

The quantum mechanical reflection and transmission coefficients R and T are assumed, as is usual in quantum mechanics, to concern exclusively the capacity of producing corpuscularlike effects, i.e., to concern exclusively the parts of the states labeled by ${}^c\lambda$'s.

A coincidence—by definition—is produced by two states ${}^c\lambda$, both carrying a localized energy fragment, massive or not.

The classification of the coincidences defined in Sec. 4.1 continues to hold, but it is now asserted concerning the states ${}^c\lambda$.

The hypothesis H_2 *is transferred to the states* ${}^{c}\lambda$.

The hypotheses H_1 *and* H_3 *are dropped.*

The values V_{1j} registered via the set of devices D_{1A} are assumed to depend on the characters of the polarizer P_2, and the values V_{2j} registered via the devices D_{2A} are assumed to depend on the characters of the polarizer P_1, for *any* coincidence, *the coincidences* $C(i, i, n, n)$ *included*, as soon as the intersections of the geometric shadow of P_1 and P_2 with, respectively, the spatial domain covered by P_2 and the spatial domain covered by P_1, are nonnull. *This condition on the geometric shadows becomes now crucial, since* $R_1 = R_2 = 0$ *ceases to ensure absence of any reflection: reflections of pure wave parts of the microsystems remain possible even if the reflections of the corpuscular parts were eliminated by the use of quasi-ideal polarizers.* This assumption will permit the model built here to produce an explicit expression of Bell's fundamentally important notion, involved *per absurdum* in his proof, that quantum mechanics asserts crossed dependences even when $R_1 = R_2 = 0$.

In the case of the coincidences $C(i, i, n, n)$, the crossed dependence is conceived to be instituted *exlusively* via the parts of the microsystem labelled ${}^{p}\lambda_{qg}$, i.e., via the reflections of pure wave parts of the microsystems. But a very important distinction comes in here between massive microsystems and photons.

If $m_0 \neq 0$, the reflected waves propagate with supraluminal phase velocity, so a coincidence $C(i, i, n, n)$ can be conceived of as involving contributions from two types of reflected entities ${}^{p}\lambda_{qg}$, namely either with $g = n$ (*same* generation), or with $g < n$ (*preceding* generation).

If $m_0 = 0$ (photonic limit), the contributions to the coincidences $C(i, i, n, n)$ which correspond to the hypothesis $g = n$ raise specific questions mentioned below. Nevertheless, in general, according to this model, inside the class of coincidences $C(i, i, n, n)$—which in the preceding model (11) did not at all contribute to the crossed dependences—the crossed dependences *do* emerge *even for photons*, via reflection of pure wave parts.

For the other three types of coincidences, the crossed dependences are conceived to be instituted in general *both* via reflections of pure wave parts (involving supraluminal velocities for de Broglie corpuscular waves) *and* via reflections of the parcels of mass-energy involved in the microsystems (travelling with infraluminal velocities for de Broglie parcels m_0c^2). As specified in Sec. 4.1, in consequence of these reflections, the crossed dependences involve, in general, different generations of pairs.

With these modifications, the representation (11), corresponding to the theoretical quantum mechanical prediction, acquires a modified form, asserting the presence, in the arguments, of a conveniently indexed reflected wave ${}^{\rho}\lambda_{qg}(P_q)$, $g=n$, *as soon as the geometry of the experiment ensures crossed dependences*:

$$\exp(V_{1j}V_{2j'})=\iint V_{1j}(P_1, {}^{ci}\lambda_{1n}, {}^{\rho}\lambda_{2n}(P_2))\, V_{2j'}(P_2, {}^{ci}\lambda_{2n}, {}^{\rho}\lambda_{1n}(P_1))$$
$$\times \pi({}^{ci}\lambda_{1n}, {}^{\rho}\lambda_{2n}, {}^{ci}\lambda_{2n}, {}^{\rho}\lambda_{1n})\, d({}^{ci}\lambda_{1n})\, d({}^{\rho}\lambda_{2n})\, d({}^{ci}\lambda_{2n})\, d({}^{\rho}\lambda_{1n})$$
$$+\iint V_{1j}(P_1, {}^{c\rho}\lambda_{2n}(P_2), {}^{\rho}\lambda_{2n}(P_2))\, V_{2j'}(P_2, {}^{c\rho}\lambda_{1n}(P_1), {}^{\rho}\lambda_{1n}(P_1))$$
$$\times \pi({}^{c\rho}\lambda_{2n}, {}^{\rho}\lambda_{2n}, {}^{c\rho}\lambda_{1n}, {}^{\rho}\lambda_{1n})\, d({}^{c\rho}\lambda_{2n})\, d({}^{\rho}\lambda_{2n})\, d({}^{c\rho}\lambda_{1n})\, d({}^{\rho}\lambda_{1n}) \tag{24}$$

As to the representation of the more general mean (12) which can emerge in the experiments, it acquires the modified form

$$\exp(V_{1j}V_{2j'})=\iint V_{1j}(P_1, {}^{ci}\lambda_{1n}, {}^{\rho}\lambda_{2g}(P_2))\cdot V_{2j'}(P_2, {}^{ci}\lambda_{2n}, {}^{\rho}\lambda_{1g}(P_1))$$
$$\times \pi({}^{ci}\lambda_{1n}, {}^{\rho}\lambda_{1g}, {}^{ci}\lambda_{2n}, {}^{\rho}\lambda_{2g})\, d({}^{ci}\lambda_{1n})\, d({}^{\rho}\lambda_{1g})\, d({}^{ci}\lambda_{2n})\, d({}^{\rho}\lambda_{2g})$$
$$+\iint V_{1j}(P_1, {}^{ci}\lambda_{1n}, {}^{\rho}\lambda_{2g}(P_2))\, V_{2j'}(P_2, {}^{c\rho}\lambda_{1g'}(P_1), {}^{\rho}\lambda_{1g}(P_1))$$
$$\cdot \pi({}^{ci}\lambda_{1n}, {}^{\rho}\lambda_{2g}, {}^{c\rho}\lambda_{1g'}, {}^{\rho}\lambda_{1g})\cdot d({}^{ci}\lambda_{1n})\, d({}^{\rho}\lambda_{2g})\, d({}^{c\rho}\lambda_{1g'})$$
$$d({}^{\rho}\lambda_{1g}) + \text{third term} + \text{fourth term} \tag{25}$$

The use, in the second term on the right-hand side of (25), of an index $g'\neq g$ stresses the fact that one has to *distinguish* in general between the generations to be assigned, respectively, to a reflected part ${}^{c\rho}\lambda$ of a microsystem, which carries a localized energy fragment, and to a reflected part ${}^{\rho}\lambda$ of a pure wave, since these two sorts of entities travel in general with different velocities. When $m_0\neq 0$, (25) can contain contributions from both indexes $g=n$, $g'=n$ *and* indexes $g<n$, $g'<n$, while when $m_0=0$, *only* contributions from indexes $g=n$ and $g'=n$ are possible.

As long as $R_1\neq 0$, $R_2\neq 0$, with (24)—as well as with (25)—Bell's proof of an inequality clearly cannot be resurrected. Then nothing hinders us from posing predictional compatibility between the representation (24) and the quantum mechanical mean, according to some convenient "discretization" model[1,12–16] of hidden spin variables.

When the photonic limit $m_0=0$ is realized and, furthermore, if it is supposed that, rigorously, $R_1=R_2=0$ (ideal polarizers), the representation (24) reduces to its first term alone. But, in contradistinction to the first term of (11), the first term of (24) does not identify with Bell's representation because of the presence of the reflected wave parts ${}^{p}\lambda_{2n}(P_2)$ and ${}^{p}\lambda_{1n}(P_1)$ in the arguments of V_{1j} and V_{2j}, respectively. However, in this case, the explanation of the possibility of crossed dependence obliges one to study a more detailed modelization of the designata indicated by the verbal pointers "a photon" and "creation of a pair of photons" (see Ref. 45, the second bibliographic entry, pp. 254–260). When an already created photon propagates freely, its velocity is Einstein's constant c. But during a process of *creation* of a pair of photons the situation is more complex. To begin with, the wave parts might be created before and after the energy bundles. Furthermore, if a reflected wave part ${}^{p}\lambda_{1n}(P_1)$ turns back upon a more recently created energy bundle ${}^{ci}\lambda_{1n}$ that advances toward P_1, and super-poses itself on this bundle, but advancing in the opposite direction, while the wave part ${}^{p}\lambda_{2n}(P_2)$ advancing towards P_1 covers the energy bundle ${}^{ci}\lambda_{2n}$ that advances toward P_2—all this possibly *before* the time when the waves surrounding these energy bundles have begun to occupy two mutually dis-connected spatial domains—what velocities and what influences do arise? We are here at the extreme limits of our present knowledge. It would be naive to make assertions, *a fortiori*, to try to calculate. We can only attempt investigations. Aspect's experiments are such an investigation, and they will have to be combined with other investigations, conceptual and experimental.

The representations (24) and (25) amount to a modelization of the quantum mechanical crossed dependences in terms of influences carried by reflected parts of the microsystems involved, *both* pure wave parts and wave parts including also a localized energy fragment.

These influences carried by reflected entities construct a type of causality not envisaged up to now, *a folded, a zigzag, a reflexive causality*. With respect to this type of causality, the sort of causality conceived of up to now appears as *a particular case of outstretched, unreflexive, one-way cusality*. So the de Broglie–Bohr nonseparability is not incompatible with causality. It only involves a causality more complex than that which we tried to associate with it.

It can be shown—but this will be done in another work—that it is possible and fruitful to distinguish between causal (or deterministic) modelizations, and operational predictability. Neither Bohr nor Einstein made this distinction explicitly. Therefrom stems an important part of their controversies as well as Bohr's concept of "essential" indeterminism.

Aspect's Experiments. Inasmuch as the employed polarizers did *not* ensure *rigorously* the condition $R_1 = R_2 = 0$ for the absence of reflection of localized quanta of energy ${}^{c}\lambda$ while the intensity of the source *was* indeed low enough for justifying reference to the theoretical quantum mechanical prediction, Aspect's results admit an obviously Einstein-separable interpretation expressed by the representation (24).

Inasmuch as the material conditions have been such that in fact neither the hypothesis $R_1 = R_2 = 0$ nor the comparison with the representation (24) are justified, only the representation (25) being in fact adequate, the results again admit the Einstein-separable interpretation expressed by (25).

Inasmuch as the condition $R_1 = R_2 = 0$ *has* been rigorously ensured, but the intensity of the source in fact has not been low enough to separate the pairs one by one, the measured results correspond to the representation (25) reduced to its first term, still clearly Einstein-separable. Each coincidence $C(i, i, n, n)$ in the first term of (24) contains reflected contributions from exclusively pure wave parts ${}^{p}\lambda_{qg}$ of the photons involved, with $g < n$, i.e., stemming from a generation of a pair which precedes the generation n of the two localized energy bundles $h\nu$ (labeled ${}^{ci}\lambda_{qn}$) by which that coincidence is produced. A pure wave-part ${}^{p}\lambda_{2g}$, $g < n$, reflected by the polarizer P_2 superposes on the state ${}^{ci}\lambda_{1n}$ incident on the polarizer P_1 and acts on it in a way that depends on the characteristics of the polarizer P_2 at the time when the reflection took place. This action influences the outcome of the interaction between P_1 and the superposition of ${}^{ci}\lambda_{1n}$ with ${}^{p}\lambda_{2g}$, and vice-versa for ${}^{ci}\lambda_{2n}$ and ${}^{p}\lambda_{1g}$ (heteromicrointerferences). This seems coherent with the Bose–Einstein statistical behavior of the ensembles of photons, with the existence of laser phenomena, etc.

Finally, inasmuch as both the condition $R_1 = R_2 = 0$ and separation of the pairs one by one have been indeed rigorously realized, we are in the case considered before, corresponding to the representation (24) reduced to its first term alone, which touches the extreme frontiers of our present knowledge and calls for a more detailed modelization.

It would be highly interesting to establish to which one of the enumerated possibilities Aspect's results actually correspond.

Random flipping of the polarizers does not modify this analysis. As long as the flippings are random, the calculation of the statistical mean of products of pairs of values $(V_{1j}, V_{2j'})$ cannot distinguish between correlations of products of simultaneous values, or correlations of products of nonsimultaneous values. And quantum mechanics predicts exclusively statistical means.

The Representation (24), (25) and Einstein Separability. The representation (24), (25), together with the analyses which led to it, define and stress the distinctions that have to be made between:

(a) existence of supraluminal influences, and violation of relativity;

(b) de Broglie–Bohr nonseparability and nonseparability as opposed to Einstein separability (which is indeed a still ill-defined concept needing further specification, but certainly not a void concept, nor incompatible with the de Broglie–Bohr nonseparability).

Words are very important. The modelization which corresponds to the representation (24), (25) shows clearly that the use of one and the same word "system" for two entities so fundamentally different as a "mobile" in Einstein's sense and a de Broglie microsystem, would be highly misleading. When it is asserted that "The real factual situation of the system S_2 is independent of what is done with the system S_1 which is spatially separated from the former,"[23] this—quite obviously—cannot be applied to entities like a de Broglie microsystem, which are *defined* such that the state of the designatum of S_2 *can* in general interact via supraluminal phase propagation with the state of the designatum of S_1. It might come out that Einstein's principle, in spite of the singular mode used in Einstein's formulation, is valid only for ensembles of de Broglie microsystems, composing a "mobile" in the macroscopic sense of the term. Language is an instrument, not a god, nor chains. And between language and thought there are subtle and strong connections. So the language must be surveyed and forged deliberately and progressively. Einstein's formulation simply did not take into account de Broglie microsystems, and these probably are the very origin of the differentiation between heavy microentities which compose the macromobiles, and photons which compose the fields. Einstein seems to have simply missed perception of this origin. Bell's theorem, as analyzed here, focuses attention upon this failure. This failure may be at the base of the obstacles which hinder a fully satisfactory unified quantum theory of heavy microsystems and of fields.

Anyhow, *the representation* (24), (25) *cannot be asserted to "violate" Einstein's concept of separability*, still partially hidden in its deficient formulation. The contribution to crossed dependence due to supraluminal reflected de Broglie waves ${}^{\rho}\lambda$ entails a sort of nonseparability which probably will be found to be fully consistent with a properly improved specification of the designatum toward which Einstein's formulation tried to point. The de Broglie-Bohr quantum non-separability will probably be found to complete Einstein's concept of separability, or even to generate an improved specification of it.

Global View. The initial representation (11), (12) was factually inacceptable. Nevertheless it sufficed for highlighting three correlated crucial points.

The quantum mechanical crossed dependences can be strongly related with the fact that the measurement devices act also as reflecting obstacles.

A modelization of the quantum mechanical crossed dependences in terms of influences carried by microscopic entities reflected by the measurement devices does not necessarily violate Einstein's principle of separability.

Bell's proof establishes exclusively that absence of crossed dependence is incompatible with quantum mechanics. It does not establish that the existence of crossed dependence is incompatible with Einstein separability, hence with Einstein's theory of relativity.

The final representation (24), (25) associates the conclusions enumerated above with a deeper analysis of the semantic content assignable to the quantum mechanical crossed dependences. It takes explicitly into account the particle-*and*-wave content of the microsystems and incorporates a drastic distinction between supraluminal phase velocities of corpuscular de Broglie waves and violation of Einstein's relativity. The more complex interpretation of the quantum mechanical crossed dependences offered by the representation (24), (25) may come out to be acceptable as "factually true," whatever these words might mean. Combined with other results concerning joint probabilities obtained by this author,[20] by Evrard,[27] and Cohen,[28] the representation (24), (25) might contribute to a finally acceptable causal reconstruction of quantum mechanics.

7. EXPERIMENTS

We have advanced above a possible interpretation of Aspect's results. In order to examine this interpretation specifically, the following explorations could be envisaged.

1. The quantum mechanical prediction depends exclusively on the angle θ between the directions $\mathbf{u}_1$ and $\mathbf{u}_2$ of the polarizers; it does not take into account the real dimensions of the reflecting surface of the polarizers. Only values refered to some unity of surface seem to be used, but with respect to what unity? Such a prediction seems to be flawed by indiscriminations stemming from an incomplete use of the available information concerning the acting factors.

Therefore it would be interesting to consider each case realistically and to study the situations of absence of *any* reflection, of absence of pure-wave reflection, as well as of reflection of localized masses. These situations are those where the angles and the reflecting surfaces involved are such that the intersections of P_1 with the geometric shadow of P_2, and of P_2 with the geometric shadow of P_1, are both zero and, furthermore, the specified shadows are far from each other (in order to minimize diffraction) (Fig. 3)

Under such conditions—if the quantum mechanical crossed dependence

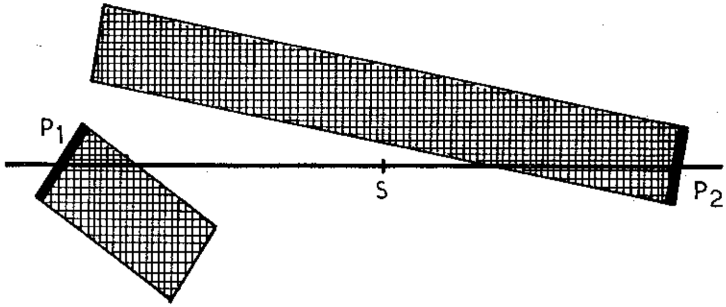

Fig. 3. Situation of absence of any reflection, absence of pure wave reflection, and absence of localized-mass reflection.

is indeed due to reflections—Bell's inequality should be verified *for any values of the quantum mechanical reflection coefficients* R_1 *and* R_2. *In particular, it should also be verified when photons and quasi-ideal polarizers are used.*

This line of research would furthermore have connections with the very significant works of Selleri,[12] Pearl,[13] Barut and Meystre,[14] Mermin,[15] and Seipp.[16]

2. Either in connection with spin measurements, or independently of these, an experimental study of the microphenomena from a corpuscular microinterference field could be attempted. Then one could pass to the photonic limit.

In the case of the spin measurements on a pair the investigation concerns the spatial domains between the source of pairs and the polarizers. In this domain each registration process creates, for a brief time, its own interference field. The birth, the characters, the evolution, the effects and the vanishing of this field could be explored via techniques like those preconized by Leggett[29] or those practiced by Chakravarty,[30] Washburn and Webb,[31] Tesche,[32] Dolan,[33] Zajonc,[34] Franson,[35] Werner,[36] Grangier and Aspect,[37] Rauch,[38] Lichte,[39] and others, involving use of the Josephson effect, of induced electromagnetic fields, of supraconductors, etc. In the case of an investigation independent of spin measurement, the polarizers could be replaced by partially reflecting mirrors.

Because of interference—self-interference, or heterointerference at the photonic limit—fluctuations of the variable mass M_0 (19) or of the energy bundle $h\nu$, respectively, might emerge. This could entail periodic alterations of "normal" thresholds (photoelectric) or effects (Compton) as a function of position and of time. *When photons are used, registration of such alterations could establish directly that "photonic heteromicrointerferences" do exist.*

If they were effectively found, such alterations would be very important. They would mark the overture to the study of the microscopic fields and of their specificities with respect to the relativistic laws identified up to now concerning other domains of facts. In particular, *the power of "communication" involved by the individual microscopic fields and by their interferences would have to be studied in new terms and by new tests, freed from Einstein's statistical concept of a "signal."*

3. Finally, quite independently of Aspect's experiments now, for *massive* particles, the values of the reflection coefficients R_1 and R_2 can be varied by varying the incident energy of S_1 and S_2 and the depths of the fields. In this way it is possible to study the relative importance and specificities of the reflections of states ${}^c\lambda$ which carry a "corpuscular" concentration of mass-energy, and of the reflections of states λ of pure corpuscular wave.

The limit $R_1 = R_2 = 0$ is crucial here again: If this limit can be approached, and if the quantum mechanical prediction is constantly verified, even with a flipping device, this would constitute

(a) a direct experimental indication of the existence of the de Broglie–Bohr quantum nonseparability, corresponding to supraluminal phase propagations of pure wave parts of de Broglie corpuscular waves;

(b) an experimental indication that in one registration of an individual result of measurement, only one de Broglie plane wave is involved, not the whole quantum mechanical wave packet (since one de Broglie wave involves a supraluminal phase velocity, while the physically significant velocities defined in quantum mechanics for a wave packet are infraluminal).

8. CONCLUSION

That quantum mechanics involves crossed dependences wich are only obscurely and weakly qualified with respect to time, is more or less known or felt by many. Lévy Leblond expressed this with particular strength,[40] d'Espagnat analyzed and illustrated it masterfully.[9] But Bell succeeded in building a formal proof involving this fact fundamentally.

The verbal elements which combine with this proof—the explicit reference to Einstein's principle and the word "locality" used for qualifying the absence of crossed dependence—have forced the quests generated by the proof into a mould of an *a priori* posed opposition between Einstein's principle of separability and the quantum crossed dependences, hence into a mould of an *a priori* posed opposition between relativity and quantum mechanics.

This incomprehensible and dramatic opposition has been a happy accident which produced a remarkable effervescence of thought. Since Wigner[1] first drew attention to Bell's theorem, and until this day, the problem defined by the theorem has been scrutinized from many different points of view. The nonclassical features of the quantum mechanical formalism, involved in this problem, were variously brought into evidence and analyzed[9–11] as much as the classical features involved.[12,14,16] Models for the quantum mechanical crossed dependences were worked out.[42,43] The theorem was criticized.[44] Arguments were expressed against the belief that Bell's theorem establishes a fundamental conflict between quantum mechanics and relativity.[45] Experiments were devised,[2,3] realized,[4–8] and analyzed.[9,10,12,46] D'Espagnat, Bonsack, Shimony, Selleri, Costa de Beauregard, Vigier (and others) maintained the debate alive. D'Espagnat developed an integrated qualification of the conceptual situation created by modern physics.[9] So, after 60 years of delay, the hidden foundations of quantum mechanics have finally been actively explored, and also amply discussed.[47–69]

This turbulent inquiry might open up a new era. A veritably relativistic unification between the field theories and the theories of heavy entities might begin, associated with investigations of the corpuscular microfields, of their self-interactions, and of *their own relativistic characters.*

The existing relativistic theories do not exhaust "relativity." Relativity is not a closed ensemble of universal dogmas; it is a *method of description*, of which the results vary when the examined object is changed.

ACKNOWLEDGMENT

The germ of this work has been a lecture given several months ago in the Seminar of Corpuscular Physics directed by Prof. B. Grossetête. I feel profoundly indebted to Prof. Grossetête for the fertile opportunity which he gave me.

I am thankful to Dr. D. Evrard for very useful remarks on two previous versions of this work.

Professor J. T. Cushing read the manuscript and made an important remark concerning the photonic limit. I am very grateful to him for this.

I am highly grateful to Professor B. d'Espagnat for having read and strongly criticized this work. However, I alone am responsible for all the conclusions.

Finally, I consider this work a homage to the memory of Louis de Broglie.

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Printed by Catherine Press, Ltd., Tempelhof 41, B-8000 Brugge, Belgium